Wind turbine rotor blades take power from the wind by slowing it down.

This is done by applying a force to the wind, and the wind applies that same force to the blades.

Objects in the path of a stream of air experience a ‘downwind’ force called drag.

The drag force was used by the earliest wind turbines. It is easy to understand how this force causes the blades to turn, but such rotors are very slow and the blades which are moving upwind actually slow the rotor down.

Drag is the force of wind pushing straight downwind.

But there is another force called ‘lift’ which always works at right angles to the wind direction.

Horizontal axis wind turbine blades never move downwind, so they can get no help from drag forces. Instead they use lift.
To create a blade design we need to specify the chord width and blade setting angle \( \beta \) at each of a series of stations along the span of the blade.

At each station we will create the right shape of the blade to produce the right loading (lift) for the 'bit of wind' with which it will have to deal.

The process of calculating the best loading and thence the best shape is known as 'finite element analysis', and it looks at what each bit of the blade needs to do.

The apparent wind which a blade 'sees' is altered by its own speed through the air.

This headwind adds to the real wind to give the apparent wind, which creates the lift and drag forces.

The headwind rotates the direction of the forces on the blade. The Drag force opposes the blade's movement. The Lift force assists the blade's movement. Both forces also push the blade downwind and slow the wind down.
The mathematics of lift and drag.

Lift and Drag forces depend on the Coefficients $C_L$ and $C_D$, which in turn depend on the cross section of blade we are using, and on the angle $\alpha$ at which the wind strikes the blade.

The chord line is the longest line in the section, joining the leading and trailing edges.

You cannot calculate the lift and drag coefficients.

They are measured experimentally in wind tunnels, and recorded in books.

Here is a typical graph of lift vs. angle of attack $\alpha$.

As $\alpha$ increases, so does the lift, until a point is reached where the blade stalls.

Most flattish objects will give a similar sort of lift/$\alpha$ curve. But cambered, streamlined sections yeild better lift/drag.
When designing a wind turbine rotor, the angle $\alpha$ will depend on the angle of the apparent wind $\phi$, and the blade angle $\beta$.

So we have control over $\alpha$, and thus control over the lift and drag produced by the blade.

We shall need to optimise the lift force, to satisfy the Betz criterion, but the blade will not work well unless the drag is minimised.

So we have to choose a section and an angle of attack, where the lift/drag ratio is high.

Finding the exact best angle $\alpha$ can be an involved process, because the lift and drag coefficients depend on both the section and the Reynolds number (a measure of the size and speed of the blade).

On the left is a pair of graphs which again relate to the NACA 4412 section for several different Reynolds numbers.

The lefthand graphs show lift/$\alpha$.

The righthand one shows lift/drag.

The straight lines through zero, represent particular lift/drag ratios.

Best lift/drag ratio for a given Reynolds number occurs where the lift/drag line is rotated as far as possible anticlockwise, so that it just touches the curve as a tangent.

For the NACA 4412, this point of contact is where $CL$ is about 1, and $\alpha$ is about 6.

Note that low Reynolds number leads to poor lift and low lift/drag ratio, which can pose problems for rotors with narrow chord widths in low winds.

There are other sections (e.g. 'ClarkY' and 'K2') which have better performance than the NACA4412 at low Reynolds number.

In practice, most sections will produce their best lift/drag at an angle of attack around 5 degrees, so as a general rule, where detailed data is not available, we can say that the blade angle $\beta$ should be set to give this angle of attack, thus:

$$\beta = \phi - 5$$
To specify blade angle $\beta$ we need to know the angle $\phi$ at which the apparent wind strikes the rotor plane.

**Blade Viewed from the Tip**

Headwind is greater near the tip (where $r=R$) than it is near the root, so the angle $\phi$ changes.

This means that the ideal shape for the blade is twisted, like this.

**Calculating the Correct Blade Setting Angle $\beta$**

$$\beta = \phi - \alpha$$

Where $\tan(\phi) = \frac{2V/3}{(r/R)\lambda V}$

$$= 2R/(3r\lambda)$$

So the blade angle $\beta$ is

$$\beta = \text{atan}(2R/3r\lambda) - \alpha$$

Where $\alpha$ is usually around 5 degrees.

More mathematics which come in useful on the next page.....

**Thrust**

$$= \text{LIFT} \cos(\phi) + \text{DRAG} \sin(\phi)$$

**Driving Force**

$$= \text{LIFT} \sin(\phi) - \text{DRAG} \cos(\phi)$$

$$= \text{LIFT} \sin(\phi) (1 - \cot(\phi)/k)$$

$$= \text{LIFT} \sin(\phi) (1 - (3r/2R)\lambda/k)$$

Where $k$ is LIFT/DRAG ratio.
Having worked out $\beta$ we still need to work out the **Chord width**. Here is the logic:

Each blade element has a certain band of wind to process.
As radius $r$ grows smaller near the centre, the amount of wind in the band gets smaller too.

The outer parts of the blade therefore do the most work. The inner part is less important but needs a different shape.

To satisfy Betz, the wind in each part of the swept area of the rotor must be slowed down to $1/3$ of its upstream velocity, and this slowing is done by the THRUST force, which is very closely related to the LIFT force.

**NEGLICING DRAG (very small error), THRUST = LIFT $\cos(\phi)$**

FOR BETZ, THRUST = $(4/9)p\Delta r^2 = (4/9)p(2\pi r \Delta r) \Delta r^2$

AND WE KNOW THAT LIFT = $CL(p/2)BC\Delta r(\text{APPARENT WIND})^2$

$=CL(p/2)BC\Delta r(\lambda V(r/r) / \cos(\phi))^2$

THIS LEADS TO A ROUGH EXPRESSION FOR THE CHORD WIDTH $C$ WHICH WILL PRODUCE THE RIGHT AMOUNT OF THRUST TO MEET THE BETZ CONDITION

$$C = \frac{16\pi R (R/r)}{9\lambda^2 B}$$

**CONCLUSIONS**

$C$ IS INVERSELY PROPORTIONAL TO RADIUS $r$.
so the blade shape should be tapered

$C$ IS INVERSELY PROPORTIONAL TO BLADE NUMBER $B$
so fewer blades will be wider blades

$C$ IS INVERSELY PROPORTIONAL TO TIP SPEED RATIO SQUARED
so doubling speed means cutting blade width down to $1/4$

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Back of envelope blade design:-

1. Choose rotor diameter $D$ to suit your power requirements

<table>
<thead>
<tr>
<th>Diameter (m)</th>
<th>(Watts) Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50-100</td>
</tr>
<tr>
<td>2</td>
<td>250-500</td>
</tr>
<tr>
<td>3</td>
<td>500-1000</td>
</tr>
<tr>
<td>4</td>
<td>1000-2000</td>
</tr>
<tr>
<td>5</td>
<td>2000-3000</td>
</tr>
</tbody>
</table>

2. Choose a tip speed ratio $\lambda$.

You are free to use is trial and error here.
I suggest you opt for a tip speed ratio between 5 and 6.

Tip speed ratio will affect rpm.

\[ \text{shaft speed} = \frac{60\lambda V}{(nD)} \text{ rpm} \]

3. Decide how many blades $B$ to use

\(B=3\text{ is the best.} \)
\(\text{Or try } B=80/\lambda^2\)

4. The width of the blade $C$ in the outer portion, will be:

\[ C = \frac{4D}{(\lambda^2 B)} \]

For example if $D=2\text{m}$, and tip speed ratio $= 7$ and $B=2$, then $C = \frac{4 \times 2}{49 \times 2} = 0.08\text{m} \text{ (or 8cm)}$.

The outer part is the most important, but the inner part should be made wider, to help with starting torque.

5. To find the best blade setting angle $\beta$, read it from this graph:

This is based on the ideal angle for a point near the tip.

Straight, untapered, untwisted blades in practice many wind turbine blades are built with constant width and constant blade angle, like this. There is surprisingly little loss of efficiency by making this compromise.

But there are other good reasons to use a twist and a taper:

Better starting

Stronger blade root
Factors affecting the power coefficient

Loss 1 is the wind which escapes around the side of the rotor. Betz figures out that the best we can do is catch 0.593 of the power, and that to catch even that much we need to slow the wind down to 1/3 of its upstream, free velocity $V$.

Loss 2 is the lost power in the swirl created by high torque rotors. Glauert figured out that this is worst at low tip speed ratios.

Loss 3 is due to the fact that we are not able to be everywhere at once. Where there are only a small number of blades, the thrust loading is higher, and some wind prefers to go around the tips. This is known as 'Tip Loss'.

Loss 5 is drag loss, which depends on LIFT/DRAG ratio. It gets worse for high tip-speed-ratio rotors, where the lift force is rotated furthest from the direction of blade movement.

Combined effect of all these losses

$$\text{DRIVING FORCE} = \text{LIFT} \sin(\phi) \left(1 - \frac{3r}{2R}\right) \lambda / k$$
where $k$ is LIFT/DRAG RATIO (see p5)
SO LIFT/DRAG MUST INCREASE WITH INCREASING TIP SPEED RATIO OR DRAG TAKES A HEAVY TOLL.
So what is the best design for a wind turbine rotor?

From the graphs, it looks as if a tip speed ratio around 5 is ideal, with as many blades as possible. The trouble with having lots of blades is that they have to be very narrow, or run at very low tip speed ratio (or both), to satisfy the Betz condition.

The perfect wind turbine rotor has an infinite number of infinitely narrow blades.

The ‘windflower’ type of rotor (right), created by Claus Nybroe at Windmission, follows this logic.

Due to the low Re-numbers the blade profile must be carefully selected and rather thin. To obtain strength and torsional stiffness, this requires a composite structure and skilled workmanship.

Here is a less ambitious planform shape for a blade:

Once you have chosen a blade planform, then the number of blades is dictated by the tip speed ratio \( \lambda \):

1 blade, \( \lambda = 9 \)

2 blades, \( \lambda = 6 \)

3 blades, \( \lambda = 5 \)

10 blades, \( \lambda = 3 \)

**THE BLADE ANGLES ARE DIFFERENT IN EACH CASE. ONLY THE PLANFORM IS THE SAME.**
High speed blades
(pros and cons)

The graph to the right shows the speeds and electrical power outputs of windmills with a range of rotor sizes, running at tip speed ratio of 5, in a 12m/s rated windspeed.

For this graph, power is calculated on the basis of rotor \( C_p = 0.25 \) and other losses=40% overall, which is easily possible for small wind turbines. (Other losses are friction, iron, copper and rectifier losses to produce the electricity output.)

Choice of rotor size (diameter) depends on power required.

Choice of tip speed ratio \( \lambda \) depends on many factors. High tip speed ratio results in higher shaft speed is more efficient for generating electricity, which often outweighs these disadvantages:

1. Noise from the blades is higher
2. Vibration in case of 2-bladed (or 1-bladed).
4. Reduced rotor efficiency, due to drag, and tip loss.
5. Starting difficulties, if the shaft is stiff to turn.

Starting torque can be estimated from the formula

\[
\text{TORQUE} = \frac{\sqrt{2} R^3}{(\text{DESIGN TIP SPEED RATIO})^2}
\]

For example a 2m diameter with tips speed ratio \( \lambda = 5 \) rotor in a 4m/s windspeed will have starting torque

\[
\text{TORQUE} = \frac{4^2 \times 2^3}{5^2} = 0.64 \text{ Nm}
\]

N.B. This is only an approximation!

Blade tips travelling at speeds in excess of 80m/s will suffer from erosion of the leading edges due to impact of small particles born by the wind. This can be countered to some degree, by the use of special tough coatings.

A rotor with tip speed ratio 7 in a 12m/s wind or a 5m diameter rotor running at 350rpm will be at risk from blade erosion.

The effect increases dramatically with increasing speed.

High tip speed ratio rotor blades will often have a strong taper.

Blade root is tapered out wide, to improve starting. Tips are tapered down to reduce noise.